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GRAVITY RECOVERY AND CLIMATE EXPERIMENT

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DOCUMENT CHANGE RECORD

Issue	Date	Pages	Change Description
01.0	Dec 1, 2003	All	Initial Version
01.1	July 24, 2004	6-16	Corrected TDT and GPS-time definitions in II.1.1; renamed sections II.1.1 and II.1.2; added clarifying text to II.2.1; updated and corrected references

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I INTRODUCTION

I. 1 PURPOSE OF THE DOCUMENT

This document serves as a record of the processing standards, models & parameters adopted for the generation of the Level-2 gravity field data products by the GRACE Science Data System component at The University of Texas Center for Space Research (UTCSR). This document is issued once for every release of Level-2 data products generated by UTCSR. That release number is included in the title of this document.

This document applies to the Level-2 GRACE products of the kind (see *Product Specification Document* or *Level-2 User Handbook*)

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This document may be used in conjunction with:

1. GRACE Product Specification Document (327-720)
2. GRACE Level-2 User Handbook (327-734)
3. GRACE GFZ L-2 Processing Standards Document (327-743)
4. GRACE JPL L-2 Processing Standards Document (327-744)
5. GRACE AOD1B Product Description Doc (327-750, GR-GFZ-AOD-0001)

I. 2 DOCUMENT CHANGE HISTORY

This document has been previously issued for the following Level-2 data product releases, in reverse chronological order:

Product Release	Date Document Issued	Remarks
none	none	

The principal changes since the previous issue of this document are described in the remainder of this section, if necessary.

I. 3 OVERVIEW OF DATA PROCESSING

This section contains a brief overview of the data processing done to obtain the Level-2 products in this release.

The gravity field estimates were made using the conventional dynamic, linear least squares adjustment for the orbit and gravity field from an optimally weighted combination of the GPS & K-Band tracking data collected by the GRACE satellites. Some specifics follow in the next table.

Processing Institution	**University of Texas Center for Space Research**	
Software Used		
Orbit Software	MSODP	Version 2003.1g
Linear Solver Software	AESoP	Version 1.4.0
GRACE Data Products Used		
Product ID & Release	*Data Rate*	*Remarks*
ACC1B (RL=00)	1 second	Used in the integrator
SCA1B (RL=00)	5 second	For observation models & transforming body-fixed accelerations
KBR1B (RL=00)	5-second Range Rate only	
GPS1B (RL=00)	30-second Double Differences between one GRACE satellite, one GPS satellite & 2 ground stations	12-station network. IGS Precise Orbits used for GPS satellites – and held fixed during analysis
AOD1B (RL=01)	Used as part of background gravity acceleration models	
Other Notes on Methodology		
Solution obtained as an optimally weighted combination of GPS double differences for each satellite and inter-satellite K-Band Range-Rate – using **one-day dynamic arcs** over the prescribed data span.		
The statistical constraints or conditioning applied during the estimation process is described in the README files accompanying the products.		

II ORBIT DYNAMICS MODELS

II. 1 EQUATIONS OF MOTION

The equations of motion for both GRACE satellites are identical in mathematical form. In the remainder of this chapter, the equations will be provided for a single Earth orbiting satellite, with the understanding that the same equations apply to both GRACE satellites. Where appropriate, the parameters or conditions unique to each satellite will be specified.

In the inertial frame

$$\ddot{\vec{r}} = \vec{f}_g + \vec{f}_{ng} + \vec{f}_{emp}$$

where the subscript "g" denotes gravitational accelerations; "ng" denotes the acceleration due to the non-gravitational or skin forces; and "emp" denotes certain empirically modeled forces designed to overcome deficiencies remaining in the force models.

II.1.1 Time Systems

The independent variable in the equations of motion is the time system TDT (Terrestrial Dynamical Time). The relationship of this abstract, uniform time scale to other time systems is well defined. The table below shows the relationship between various time systems and the contexts in which they are used.

System	Relations	Notes	Standards
TAI	Fundamental time system	International Atomic Time	n/a
UTC	UTC = TAI – n1 (Time-tag for saving intermediate products)	n1 are the Leap Seconds	Tables from USNO
TDT	TDT = TAI + 32.184s	This is the independent variable for numerical integration.	IAU 1976 Recommendation (equivalent to using TT in *IERS(2003)* standards)
GPS	GPS = TAI – 19s (time-tag of GRACE observations)	Relationship between GPS & TDT is fixed at 19 seconds	Time-tags in sec since 1200 Jan 01, 2000 GPS Time.

II.1.2 Reference Frames

The fundamental reference frame for the mathematical model is the non-rotating, freely-falling (inertial) reference frame, with the origin defined as the center of mass of the Earth. The Inertial and Earth-fixed reference frames, and their relative orientations and associated standards are further described in the chapter on Earth Kinematics.

II. 2 GRAVITATIONAL ACCELERATIONS

The gravitational accelerations are the sum of direct planetary perturbations and the geopotential perturbations. The vector of direct planetary perturbations is evaluated using the planetary ephemerides. The geopotential itself is represented in a spherical harmonic series with time-variable coefficients, to a specified maximum degree and order, and accelerations are computed by evaluating the Earth-fixed gradient of the geopotential. The accelerations are then rotated (after summation with the non-gravitational accelerations) to inertial frame for the integration of equations of motion. In general,

$$\vec{f}_g = {}_{3x3}M_{ef}^{in}(P,N,R)\,\vec{f}_g^{\,ef}$$

The 3x3 rotation matrix M, which depends on Earth orientation is described in the chapter on Earth Kinematics.

Contributions to the spherical harmonic coefficients of the geopotential, and the associated implementation & standards are now compiled. The geopotential at an exterior field point, at time t, is expressed as

$$U_s(r,\phi,\lambda,t) = \frac{GM_e}{r} + \frac{GM_e}{r}\sum_{l=2}^{N_{\max}}\left(\frac{a_e}{r}\right)^l \sum_{m=0}^{l} \overline{P}_{lm}(\sin\phi)\left[\overline{C}_{lm}(t)\cos m\lambda + \overline{S}_{lm}(t)\sin m\lambda\right]$$

where r is the geocentric radius, and (ϕ,λ) are geographic latitude and longitude, respectively, of the field point.

The model used for propagation of the equations of motion of the satellites is called the Background Gravity Model. This concept, and its relation to GRACE estimates, is described further in the *Level-2 User Handbook*. The details of the background gravity model are provided here.

II.2.1 Mean Geopotential & Secular Changes

Parameter	Value	Remarks
GM_e	3.986004415E+14	*IERS(2003) Standards (value is consistent with using TDT or TT as the time argument)*
a_e	6378136.3 m	
$N_{\max}$	complete to degree and order 200	***GGM01C*** is background static model (*Tapley et al. 2004)*
$\dot{\bar{C}}_{20}$	+0.116276E-10/y	*IERS(2003) Standards (Epoch of C_{20} is 2000.0)*
$\dot{\bar{C}}_{21}$ & $\dot{\bar{S}}_{21}$	-0.33745E-11/y +0.16059E-10/y	*IERS(2003) Standards (Epoch of C_{21} and S_{21} is 2000.0)*
Note 1: The normalization conventions are as defined in *IERS(1996)*, Chapter 6, Eqs 2-3. Note 2: The implementation of computation of spherical harmonics and its derivatives is as described in (*Lundberg and Schutz, 1988*). Note 3: The degree 1 terms are exactly zero in the geopotential model.		

II.2.2 Solid Earth Tides

Solid Earth tidal contribution to the geopotential are computed approximately as specified in Chapter 7, *IERS(1992) Conventions*. Corrections to specific spherical harmonic coefficients are computed and added to the mean field coefficients.

Model	Notes	
Planetary Ephmerides	DE-405	
Frequency Independent Terms	Degree 2 & 3 – expression in Eq. (1), Ch.6, *IERS(2003)*.	Constants from *IERS(1992)* are used.
	Ellipticity contributions from Degree 2 tides to Degree 4 terms	Included
	Anelasticity Contributions	Not included
Frequency Dependent Terms	Tidal corrections to (2,1)	• 35 principal tide lines, as per *Wahr (1981)* for model 1066A. • FCN resonance correction included in diurnal band
	Anelasticity Contributions	Not included
Permanent Tide in $\bar{C}_{20}$	4.173E-9	Subtracted from these contributions (is implicitly included in value of the mean C20)

II.2.3 Ocean Tides

The ocean tidal contributions to the geoptoential are computed as specified in Chapter 6, *IERS(1996) Conventions*, Eqs 8, 10a & 10c (with appropriate sign correction for Eq 10c). Corrections to specific spherical harmonic coefficients of arbitrary (selectable) degree and order are computed and added to the mean field coefficients.

Model	Description	Notes
Tidal Arguments & Amplitudes/Phases	*Doodson (1921)* *Cartwright & Tayler (1971)*	
Tidal Harmonics	Multi-satellite selection of harmonics for discrete tidal lines from **CSR 4.0** Coefficients *(Eanes 2002)*	Selection approach based on *Casotto 1989*.

II.2.4 Tabular Atmosphere & Oceanic Variability

The non-tidal variability in the atmosphere and oceans is removed through using the AOD1B product. This product is a combination of the ECMWF operational atmospheric model and a barotropic ocean model driven with this atmospheric model. The details of this product and its generation are given in the *AOD1B Description Document (GRACE 327-750)*.

This component of the geopotential is ingested as 6 hourly time series to degree and order 100. The value of the harmonics at intermediate epochs is obtained by linear interpolation between the bracketing data points. Prior to its use, an estimate of the atmospheric S2 tidal effects on $\overline{C}_{22}$ and $\overline{S}_{22}$ are removed from the AOD1B product. This estimate is simply the difference of the TEG4 multi-satellite estimate of this tidal harmonic and the altimetric determination of this harmonic from the CSR 4.0 tidal model. In this way, the combination of the atmospheric and oceanic S2 tidal effects on the (2,2) harmonics are modeled using the ocean tide model.

II.2.5 Rotational Deformation (Pole Tide)

The rotational deformation forces are computed as additions to spherical harmonic coefficients $\overline{C}_{21}$ and $\overline{S}_{21}$, from an elastic Earth model, as specified in Chapter 6, *IERS(1996)* Standards.

Model	Description	Notes
Elastic Earth Model Contribution to C21 & S21	Scaled difference between epoch pole position and mean pole. See Chapter III (Earth Kinematics) for the linear variation model for the mean pole.	
Polar Motion	Tabular input	*IERS C04*
Mean Polar Motion & Rates	Linear trend	*IERS(2003)*
Constant Parameters	Scale Factor = -1.2197×10^{-9} *per arc* sec	
Anelasticity	Ignored	

II.2.6 N-Body Perturbations

Unlike the geopotential accelerations, the perturbations due to the Sun, Moon and all the planets are directly computed as accelerations acting on the spacecraft. The direct effects of the objects on the satellite are evaluated using point-mass attraction formulas. The indirect effects due to the acceleration of the Earth by the planets are also modeled as point-mass interactions. However, for the Sun and Moon, the indirect effects include, in addition, the interaction between a point-mass perturbing object and an oblate Earth – the so-called Indirect J2 effect.

Model	Description	Notes
Third-Body Perturbation	Direct & Indirect terms of point-mass 3rd body perturbations	
Indirect J2 Effect	Sun & Moon only	
Planetary Ephemerides	DE-405	

II.2.7 General Relativistic Perturbations

The general relativistic contributions to the accelerations are computed as specified in Chapter 10 of the *IERS(2003) Conventions*.

II. 3 NON-GRAVITATIONAL FORCES

The nominal approach is to use the GRACE accelerometer data to model the non-gravitational forces acting on the satellite.

The model used is:

$$\vec{f}_{ng} = q \quad \left[\vec{b} + {}_{3x3} \cdot \vec{f}_{acc} \right]$$

where the q/operator represents rotations to inertial frame using the GRACE Attitude Quaternion product; b represents an empirical bias vector; and the 3x3 matrix E contains the scale factors along the diagonal, and cross-coupling terms in the off-diagonal.

The bias vector and scale matrix operate on the GRACE Accelerometer observation product, and are estimatable parameters.

II. 4 EMPIRICAL FORCES

For this product release, no empirical accelerations are modeled or estimated.

II. 5 NUMERICAL INTEGRATION

The Predictor-Corrector formulation of the Krogh-Shampine-Gordon, second order, fixed-step, fixed-mesh/order integrator is implemented.

Model	Description	Notes
Dependent Variables 1. Equations of motion (position/velocity for each satellite) 2. State Transition Matrix (position/velocity mapping terms only)		
Formulation	Cowell Formulation	
Order	7	
Step-Size	5 second	

III EARTH & SATELLITE KINEMATICS

III. 1 EARTH ORIENTATION

Earth Orientation here refers to the model for the orientation of the Earth-fixed reference relative to the Inertial reference. The former are necessary for associating observations, models and observatories to the geographic locations; and the latter for dynamics, integration & ephemerides.

Frame	System	Realization
Inertial	ICRS	J2000.0 (IERS)
Earth-fixed	CTRS	ITRF2000

The rotation between the Inertial and Earth-fixed frames is implemented as:

$$_{3x3}M_{trs}^{crs} = PNRW$$

which converts the column array of components of a vector in the terrestrial frame to a column array of its components in the inertial frame. Each component matrix is itself a 3x3 matrix, and is now individually described.

Option 1 offered in the *IERS(1996) Conventions* (Chapter 5) is implemented.

In the following, R_1, R_2, R_3 refer to the elementary 3x3 rotation matrices about the principal directions X, Y and Z, respectively.

III.1.1 Precession (P)

Following *IERS(1996)*, the IAU 1976 Precession is modeled as

$$P = R_3(\zeta_A)R_1(\theta_A)R_3(z_A)$$

where the component angles are evaluated using formulas in USNO Circular 163, Page A2. Reference epoch 2000.0 is used. The independent variable is TT since epoch J2000.0 (noon, 01-Jan-2000).

III.1.2 Nutation

Following *IERS(1996)*, the IAU 1980 Nutation model is used along with the associated corrections, such that

$$N = R_1(\Delta\varepsilon_A)R_3(\Delta\psi)R_1(\varepsilon_A + \Delta\varepsilon_A)$$

The calculation of the nutation angles & their corrections is now summarized.

Quantity	Model	Notes
Obliquity of Ecliptic (ε_A)	Polynomial	USNO Circular 163, Page A3
Nutations in Longitude or Right Ascension ($\Delta\psi$) & Obliquity ($\Delta\varepsilon$)	Linear interpolation of nutations in DE405 (IAU 1980)	
Nutation Corrections	Planetary corrections: 25 largest terms	*(Souchay 1983)*
	An-elastic corrections	Not implemented

III.1.3 Sidereal Rotation (R)

This rotation is implemented as

$$R = R_3(\theta GST)$$

where the Greenwich Apparent Sidereal Time (GST) is calculated as the sum of Greenwich Mean Sidereal Time (GMST) and equatorial components of precession and nutation. The GMST calculation uses UT1 as its independent argument, whose evaluation is also summarized in this table.

Quantity	Model	Notes
GMST	Polynomial with UT1 as independent variable.	USNO Circular 163, Page A3
Equatorial components of precession & nutation	*(Aoki & Kinoshita 1983)*	*IERS(1996)*
UT1	cubic interpolation of tabular UT1 corrections	*IERS C04*
	Diurnal tidal variations adapted from *Ray 1995* eight constituent model.	*IERS(1996) Standards*
	Nutation Corrections – 25 largest corrections to IAU 1980.	*Souchay (1983)*
NOTE: Option-1 described in *IERS(1996)* is implemented.		

III.1.4 Wobble (W)

The Polar Motion component of rotation is implemented as

$$W = R_1(y_p)R_2\left(x_p\right)$$

Quantity	Model	Notes
Tabular variations	Cubic interpolation	*IERS C04*
Ocean Tidal Variations (Diurnal/Semi-Diurnal)	Orthoweights Formulation	*IERS(1996) Standards*
Note 1: The rotation matrices are implemented in the small angle, skew-symmetric matrix formulation. Note 2: Rotational deformation accelerations & kinematic station displacements are proportional to the difference between this time-series and a linear model for the pole.		

III.1.5 Rotation of Velocity Components

The position rotations are specified in Section III. 1. The components of the velocity vector are rotated using the matrix approximation

$$\vec{v}_{crs} = M_{crs}^{trs}\vec{v}_{trs} + \left(PN\dot{R}S\right)\vec{r}_{trs}$$

III. 2 STATION COORDINATES

This section summarizes the models for the mean and time-variable parts of the station coordinates adopted for data processing.

Quantity	Model	Notes
Mean Station Positions	ITRF2000	Refers to the position of a geodetic marker at each site.
Station Velocities	Individual Station Velocities in ITRF-2000	*IERS(1996) Standards*
Ocean Tidal Loading (Diurnal/Semi-Diurnal band)	Orthoweight Formulation. Orthoweights adjusted to Scherneck model	*IERS(1996) Standards*
Station Eccentricities	Individual observation models	
Luni-Solar Solid Earth	Chapter 7, *IERS(1996)*	

Tidal displacement	(Luni-Solar ephemerides from DE-405)	
Rotational Deformation	Scaling of difference of Wobble values from a linear trend model.	See Wobble rotation or Mean Field
Tidal Geonceter (Diurnal/Semi-Diurnal)	Included within Ocean Tidal Loading model	
Atmospheric Loading	Not modeled	
Post-glacial Rebound	Not modeled	
Slow (seasonal) Geocenter Variations	Not modeled	

III. 3 SATELLITE KINEMATICS

The Science Reference Frame (SRF - see *Product Specification Document*) is used in all instances where a satellite-fixed reference frame is needed. The inertial orientation of the spacecraft (i.e. the SRF) is modeled using tabular input quaternion from product SCA1B (*ibid.*). The same product is also used for rotating the accelerometer data to inertial frame prior to numerical integration; for making corrections to the ranging observations due to offset between the satellite center of mass & the antenna location; as well as for computing the non-gravitational forces (if necessary).

At epochs where the GRACE quaternion product is not available, linear interpolation between adjacent values is used.

REFERENCES

Aoki, S, and H. Kinoshita, Note on the relation between the equinox and Guinot´s nonrotating origin, *Celest. Mech.*, **29**, pp. 335-360, 1983.

Cartwright, D. E. and R. J. Tayler, New Computations of the Tide-generating Potential, *Geophys. J. Roy. Astron. Soc.*, **33**, pp. 253-264.

Casotto, S., *Nominal Ocean Tide Models for TOPEX Precise Orbit Determination,* Ph.D. Thesis, The University of Texas at Austin, 1989.

Doodson, A.T.,The harmonic development of the tide-generating potential, *Proc. R. Soc. A.*, **100**, pp. 305-329, 1921.

Eanes, R., *The CSR 4.0 Global Ocean Tide Model* is available at ftp://www.csr.utexas.edu/pub/tide.

Lundberg, J. B., and B. E. Schutz, Recursion formulas for Legendre function for use with nonsingulargeopotential models, *Journal of Guidance, Control and Dynamics*, **11**, 31, 1988.

McCarthy, D., *IERS Conventions (1992)*, IERS Technical Note 13.

McCarthy, D., *IERS Conventions (1996)*, IERS Technical Note 21.

McCarthy, D., *IERS Conventions (2003)*, IERS Technical Note 32.

Ray, R., D. Steinberg, B. Chao, D. Cartwright, Diurnal and semidiurnal variations in the Earth´s rotation rate induced by ocean tides, *Science*, **264**, pp. 830-832, 1994.

Schwiderski, E., Atlas of ocean tidal charts and maps, part I: The semidiurnal principal lunar tide M_2, *Marine Geodesy*, **6**, pp. 219-256, 1983.

Souchay, J, M. Feissel, C. Bizouard, N. Capitaine and M. Bougeard, *Astron. Astrophys. Sup.*, **299**, 277, 1983.

Tapley, B. D., S. Bettadpur, M. M. Watkins and Ch. Reigber, The Gravity Recovery and Climate Experiment: Mission Overview and Early Results, *Geophys. Res. Lett.*, **31**, L09607, doi:10.1029/2004GL019920, 2004.

Wahr, J., The forced nutations of an elliptical, rotating, elastic and oceanless Earth, *Geophys. J. R. Astro. Soc.*, 64, pp. 705-727, 1981.